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DYNAMIC RESPONSE TO MOVING CONCENTRATED LOADS OF NON-UNIFORM SIMPLY SUPPORTED PRE-STRESSED BERNOULLI-EULER BEAM RESTING ON BI-PARAMETRIC SUBGRADES

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ABSTRACT: This paper investigates the dynamics response to non-uniform simply supported prestressed Bernoulli-Euler beam resting on bi-parametric subgrades, in particular, Pasternak subgrades and traversed by concentrated moving loads. The solution technique is base on the Galerkin Method and a modification of the Struble's technique. The deflection of the prestressed beam is calculated for several values of foundation modulus K, shear modulus G and axial force N and shown graphically. It is found that as the foundation modulus increases with fixed values of shear modulus and axial force, the displacement response of the beam decreases. Also, as the shear modulus increases with fixed values of axial force and foundation modulus, results show that the deflection of the beam model decreases. Finally, the response amplitudes of the beam model decreases with increases in the values of axial force for fixed values of shear modulus and foundation modulus . It was also observed that higher values of shear modulus is required for a more noticeable effect than that of the foundation modulus . Further more, the moving force solution is not an upper bound for an accurate solution of the moving mass problem.

Keywords: Non-uniform beam, Pasternak subgrades, Axial force, shear modulus, foundation modulus, moving force, moving mass, Resonance

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1. Introduction

In recent years, considerable attention has been given to the response of elastic beams on an elastic foundation which is one of the structural engineering problems of theoretical and practical interest. The structures Engineers in this circumstance faces the non-trivial problem posed by the singularity in the inertia of the system, a singularity which depends on spatial and time variables, consequently the problem did and still continues to attract the attention of researchers, Engineers and scientist. The problem of analysing the behaviour of a uniform elastic beam resting on Winkler foundation under the influence of a moving load has been studied in various field of engineering, applied mathematics as well as applied physics. Over the years, this moving load problem has attracted much attention of a large number of investigators [1 – 10]. As a matter of fact, there are many designs involving moving loads in one form or the other. An extensive review of moving load problems has been reported by Frybal [10] in his excellent monograph. However, work on the dynamic

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response of non-uniform elastic beam resting on bi-parametric sub-grades under the influence of moving concentrated loads is scanty. This is perhaps due to the facts, unlike the case of a uniform beam, the beam's properties such as length does not vary with span L of the beam. Nonetheless, the vibration of non uniform beam is of practical importance. For instance the cross section of some structural members such as bridge, girders, hull of ships, concrete slab etc, vary from one point to another along the structural member.

Furthermore, Gbadeyan and Idowu [11] study the dynamic response to moving concentrated masses of elastic plates on a non-Winkler elastic foundation. Oni [12] considered flexural motions of a uniform beam under the actions of concentrated mass travelling with variable velocity. Abu [13] considered the dynamic response of a Double uniform Euler-Bernoulli beam due to a moving constant load.

In a more recent time, many researchers like Oni and Ogunbamike [14], Liu and Chang [15], Oni and Omolofe [16] and Gbadeyan et al [17] had tremendously work on the dynamics of elastic systems under moving loads. This paper is concerned with the dynamic response to moving concentrated load of non-uniform simply supported pre-stressed Bernoulli-Euler beam resting on bi-parametric sub-grades, in particular, Pasternak sub-grades.

2. FORMULATION OF THE PROBLEM

Consider a structure whose displacement is given by the equation [10]

$$\frac{\partial^2}{\partial x^2}\left(EI(x)\frac{\partial^2 V(x,t)}{\partial x^2}\right)+\mu(x)\frac{\partial^2 V(x,t)}{\partial t^2}-N\frac{\partial^2 V(x,t)}{\partial x^2}$$
$$+kV(x,t)-G\frac{\partial^2 V(x,t)}{\partial x^2}=P_f(x,t)[1-\frac{1}{g}\frac{dV(x,t)}{dt}] \qquad (2.1)$$

This is the case if the structure is a non-uniform beam under tensile stress resting on Pasternak subgrades executing flexural vibration according to the simple Bernoulli-Euler theory of flexure.

The following symbols have been used in equation (2.1)

V(x, t) is the transverse displacement,

$P_f(x,t)$ is the moving force

N is the constant axial force

I(x) is the variable moment of inertia

$\mu(x)$ is the variable mass per unit length of the beam

EI(x) is the variable flexural rigidity of the beam

K is the foundation modulus

$\frac{d}{dt}$ is the substantive acceleration operator

g is the acceleration due to gravity.

G is the shear modulus

and x and t are respectively spatial and time coordinates.

The structure under consideration is simply supported. Thus, the boundary conditions are

$$V(0,t)=0=V((L,t) \qquad (2.2a)$$

$$\frac{\partial^2 V(0,t)}{\partial X^2}=0=\frac{\partial^2 V(L,t)}{\partial X^2} \qquad (2.2b)$$

While the associated initial conditions of the motion are

$$V(x,0) = 0 = \frac{\partial V(x,0)}{\partial t} \tag{2.3}$$

Furthermore, the operator $\frac{d}{dt}$ used in (2.1) is defined as

$$\frac{d}{dt} = \frac{\partial^2}{\partial t^2} + \frac{2c\partial^2}{\partial x \partial t} + \frac{c^2 \partial^2}{\partial x^2} \tag{2.4}$$

while the moving force $P_f(x,t)$ acting on the beam is chosen as

$$P_f(x,t) = Mg\delta(x-ct) \tag{2.5}$$

By substituting equations (2.4) and (2.5) into (2.1) one obtains

$$\begin{aligned} &\frac{\partial^2}{\partial x^2}\left(EI(x)\frac{\partial^2}{\partial x^2}(x,t)\right) + \mu(x)\frac{\partial^2 V}{\partial t^2}(x,t) - \mu\frac{\partial^2 V}{\partial x^2}(x,t) - G\frac{\partial^2 V}{\partial x^2}(x,t) \\ &+ KV(x,t) + M\delta(x-ct)\left[\frac{\partial^2}{\partial t^2} + \frac{2c\partial^2}{\partial x\partial t} + \frac{c^2\partial^2}{\partial x^2}\right]V(x,t) = Mg\delta(x-ct) \end{aligned} \tag{2.6}$$

As an example [10], let the variable moment of inertia and the variable mass per unit length of the beam be defined respectively as

$$I(x) = I_0\left(1 + Sin\frac{\pi x}{L}\right)^3 \text{ and } \mu(x) = \mu_0\left(1 + Sin\frac{\pi x}{L}\right) \tag{2.7}$$

Where I_0 and μ_0 are constants moment of inertia and constant mass per unit length of the corresponding uniform beam respectively. By expanding the first term of equation (2.6) and substituting equation (2.7), after some simplification and re-arrangement, the equation of motion can be written as

$$\begin{aligned} &R_0[(\frac{5}{2} + \frac{15}{4}\sin\frac{\pi x}{L} - \frac{1}{4}\sin\frac{3\pi x}{L} - \frac{3}{2}\cos\frac{2\pi x}{L})\frac{\partial^4 V(x,t)}{\partial x^4} \\ &+ (\frac{9\pi^2}{4L^2}\sin\frac{3\pi x}{L} + \frac{15\pi^2}{4L^2}\sin\frac{\pi x}{L} - \frac{6\pi^2}{L^2}\cos\frac{2\pi x}{L} - \frac{3}{2}\cos\frac{2\pi x}{L})\frac{\partial^2 V(x,t)}{\partial x^2}] \\ &+ (1 + \sin\frac{\pi x}{L})\frac{\partial^2 V(x,t)}{\partial x^2} - H_1\frac{\partial^2 V(x,t)}{\partial x^2} + H_2 V(x,t) \\ &+ \frac{M}{\mu_0}[\delta(x-ct)\frac{\partial^2 V(x,t)}{\partial t^2} + 2c\delta(x-ct)\frac{\partial^2 V(x,t)}{\partial x\partial t} + c^2\delta(x-ct)\frac{\partial^2 V(x,t)}{\partial x^2}] \\ &= P\delta(x-ct) \end{aligned} \tag{2.8}$$

Where

$$R_0 = \frac{EI_0}{\mu_0}, \quad H_1 = \frac{N+G}{\mu_0}, \quad H_2 = \frac{K}{\mu_0}, \; \& \quad P = \frac{Mg}{\mu_0} \tag{2.9}$$

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Equation (2.8) is a non-homogeneous partial differential equation with variable coefficient. Evidently, the method of separation of variables is inapplicable as a difficulty arises in getting separate equation whose functions are functions of a single variable.

3.0 METHOD OF SOLUTION

As a result of the foregoing difficulty, an approximate solution is sought. One of the approximate methods best suited for solving diverse problems in dynamics of structures is the Galerkin's method [10]. This method requires that the solution of equation (2.8) takes the form

$$V_n = \sum_{m=1}^{n} W_m(t) Y_m(x) \tag{3.1}$$

Where $Y_m(x)$ is chosen such that all the boundary conditions (2.2a) and (2.2b) are satisfied. Equation (3.1) when substituted into equation (2.8) yields

$$\begin{aligned}
&\sum_{m=1}^{n} \Bigg\{ R_0 \Bigg[\left(\frac{5}{2} + \frac{15}{4L^2} Sin\frac{\pi x}{L} - \frac{1}{4} Sin\frac{3\pi x}{L} - \frac{3}{2} Cos\frac{2\pi x}{L} \right) W_m(t) Y_m^{iv}(x) \\
&+ \left(\frac{9\pi^2}{4L^2} Sin\frac{3\pi x}{L} - \frac{15\pi^2}{4L^2} Sin\frac{\pi x}{L} + \frac{6\pi^2}{L^2} Cos\frac{2\pi x}{L} \right) W_m(t) Y_m^{11}(x) \Bigg] \\
&+ (1+) Sin\frac{\pi x}{L} \ddot{W}_m(t) Y_m(x) - H_1 W_m(t) Y_m^{11}(x) + H_2 W_m(t) Y_m(x) \\
&+ \frac{H_2}{\mu_0} \big[\delta(x-ct) \ddot{W}_m(t) Y_m(x) + 2c\delta(x-ct) \dot{W}_m(t) Y_m^{1}(x) \\
&+ c^2 H_2 \delta(x-ct) W_m(t) Y_m^{11}(x) \Big] \Bigg\} - P\delta(x-ct) = 0
\end{aligned} \tag{3.2}$$

In order to determine $W_m(t)$, it is required that the expression on the left hand side of equation (3.2) be orthogonal to the function $Y_k(x)$. Hence

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$$\int_0^L \sum_{m=1}^{n} \left\{ R_0 \left[\left(\frac{5}{2} + \frac{15}{4L^2} Sin\frac{\pi x}{L} - \frac{1}{4} Sin\frac{3\pi x}{L} - \frac{3}{2} Cos\frac{2\pi x}{L} \right) W_m(t) Y_m^{iv}(x) \right.\right.$$

$$+ \left(\frac{9\pi^2}{4L^2} Sin\frac{3\pi x}{L} - \frac{15\pi^2}{4L^2} Sin\frac{\pi x}{L} + \frac{6\pi^2}{L^2} Cos\frac{2\pi x}{L} \right) W_m(t) Y_m^{11}(x) \Bigg]$$

$$+ \left(1 + Sin\frac{\pi x}{L} \right) \ddot{W}_m(t) Y_m(x) - H_1 W_m(t) Y_m^{11}(x) + H_2 W_m(t) Y_m(x)$$

$$+ \frac{H_2}{\mu_0} \left[\delta(x-ct) \ddot{W}_m(t) Y_m(x) + 2c\delta(x-ct) \dot{W}_m(t) Y_m^1(x) \right.$$

$$\left. + c^2 H_2 \delta(x-ct) W_m(t) Y_m^{11}(x) \right] \Big\} Y_k(x) dx = P \int_0^L \delta(x-ct) Y_k(x) dx \qquad (3.3)$$

Since our elastic system has simple supports at the edges x=0 and x=L,

we choose

$$Y_m(x) = Sin\frac{m\pi x}{L} \qquad (3.4)$$

and

$$Y_k(x) = Sin\frac{K\pi x}{L} \qquad (3.5)$$

The Dirac-delta function is defined as

$$\delta(x-ct) = \frac{1}{L} + \frac{2}{L} \sum_{n=1}^{\infty} Cos\frac{n\pi ct}{L} Cos\frac{n\pi cx}{L} \qquad (3.6)$$

Substituting (3.4), (3.5) and (3.6) into (3.3). After some simplification and rearrangement and ignoring the summation sign to obtains

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$$J_1(m,k)+J_{33}(m,k)\ddot{W}_m(t)+[R_0\left(\frac{5m^4\pi^4}{2L^4}J_1(m,k)+\frac{15m^4\pi^4}{4L^4}J_{33}(m,k)-\frac{m^4\pi^4}{4L^4}J_{49}(m,k)\right.$$
$$\left.-\frac{3m^4\pi^4}{2L^4}J_{65}(m,k)-\frac{9m^2\pi^4}{2L^4}J_{49}(m,k)-\frac{6m^2\pi^4}{L^4}J_{65}(m,k)+\frac{15m^2\pi^4}{4L^4}J_{33}(m,k)\right)$$
$$+H_1\frac{m^2\pi^2}{L^2}J_1(m,k)+\Lambda H_2J_1(m,k)]W_m(t)+\forall L[\frac{1}{L}J_1(m,k)+\frac{2}{L}\sum_{n=0}^{\infty}Cos\frac{n\pi ct}{L}J_{17}(n,m,k)\ddot{W}_m(t)$$
$$+2c(\frac{m\pi}{L^2}J_5(m,k)+\frac{2}{L}\sum_{n=0}^{\infty}Cos\frac{n\pi ct}{L}(\frac{m\pi}{L})J_{21}(n,m,k))\dot{W}_m(t)$$
$$+c^2(-\frac{m^2\pi^2}{L^2}.\frac{1}{L}J_1(m,k)+\frac{2}{L}\sum_{n=0}^{\infty}Cos\frac{n\pi ct}{L}(-\frac{m^2\pi^2}{L^2})J_{17}(n,m,k))W_m(t)]=PSin\frac{k\pi ct}{L} \quad (3.7)$$

Where

$$J_1(m,k)=\int_0^L Cos\frac{k\pi x}{L}Sin\frac{n\pi x}{L}dx \quad (3.8)$$

$$J_5(m,k)=\int_0 Cos\frac{k\pi x}{L}Sin\frac{m\pi x}{L}dx \quad (3.9)$$

$$J_{17}(n,m,k)=\int_0^L Cos\frac{n\pi x}{L}Sin\frac{m\pi x}{L}Sin\frac{k\pi x}{L}dx \quad (3.10)$$

$$J_{21}(n,m,k)=\int_0^L Cos\frac{n\pi x}{L}Cos\frac{k\pi x}{L}Sin\frac{m\pi x}{L}dx \quad (3.11)$$

$$J_{33}(m,k)=\int_0 Sin\frac{k\pi x}{L}Sin\frac{m\pi x}{L}dx \quad (3.12)$$

$$I_{49}(m,k)=\int_0^L Sin\frac{3\pi x}{L}Sin\frac{k\pi x}{L}Sin\frac{m\pi x}{L}dx \quad (3.13)$$

$$I_{65}(m,k)=\int_0^L Cos\frac{2\pi x}{L}Sin\frac{k\pi x}{L}Sin\frac{m\pi x}{L}dx \quad (3.14)$$

By evaluating the integrals and substituting the results into (3.7) after some simplification and rearrangement to obtain

$$B_{11}\ddot{W}_m(t)+B_{12}W_m(t)$$
$$+\forall L\{B_{13}\ddot{W}_m(t)+B_{12}W_m(t)+B_{14}\dot{W}_m(t)-B_{15}W_m(t)\}=PSin\frac{k\pi ct}{L} \quad (3.15)$$

Where

$$B_{11}=\frac{L}{2}+\frac{L}{4\pi}A_{11} \quad (3.16)$$

$$B_{12}=R_0\left(\frac{5m^4\pi^4}{4L^3}+\frac{15m^2\pi^3}{16L^3}(1+m^2)A_{11}-\frac{m^2\pi^4}{4L^4}(9+m^2)A_{12}\right)$$
$$+H_1\frac{m^2\pi^2}{2L}+\frac{LH_2}{2} \quad (3.17)$$

$$B_{13}=\frac{1}{L}+\frac{L}{8}\left(2Sin\frac{m\pi ct}{L}Sin\frac{k\pi ct}{L}\right) \quad (3.18)$$

$$B_{14}=\frac{2cm\pi}{L}\left[\frac{2k}{\left(k^2-m^2\right)\pi}-\frac{4k}{\pi}\sum_{n=0}^{\infty}Cos\frac{n\pi ct}{L}\frac{\left(n^2+m^2-k^2\right)}{\left((n+m)^2-k^2\right)\left((n-m)^2-k^2\right)}\right] \quad (3.19)$$

$$B_{15}=\frac{c^2m^2\pi^2}{L^2}[\frac{1}{2}+\frac{1}{2}(2Sin\frac{m\pi ct}{L}Sin\frac{k\pi ct}{L})] \quad (3.20)$$

$$A_{11}=\frac{(-1)^{(1+2m)}-1}{1+2m}+\frac{(-1)^{(1-2m)}-1}{1-2m}+4 \quad (3.21)$$

$$A_{12}=\frac{L}{3\pi}+\frac{L}{4\pi}\frac{(-1)^{(3+2m)}-1}{3+2m}+\frac{(-1)^{(3-2m)}-1}{33-2m} \quad (3.22)$$

Equation (3.15) is the fundamental equation of our problem. Two special cases of (3.15) are discussed below.

Case1: Simply supported non-uniform Bernoulli Euler beam transverse by moving force.

When we set $\forall = 0$ in (3.15), we obtain response of the simply supported non-uniform Bernoulli-Euler beam subjected to a moving force. Equation (3.15) reduces to

$$B_{11}\ddot{W}_m(t) + B_{12}W_m(t) = PSin\frac{k\pi ct}{L} \tag{3.23}$$

Equation (3.23) can be re-written as

$$\ddot{W}_m(t) + \beta_f^2 W_m(t) = \frac{P}{B_{11}} Sin\frac{k\pi ct}{L} \tag{3.33}$$

where

$$\beta_f^2 = \frac{B_{12}}{B_{11}} \tag{3.34}$$

Solving equation (3.33) in conjunction with the initial conditions, we obtains the expression for $W_m(t)$ as

$$W_m(t) = \frac{P}{2B_{11}}\left(\frac{\beta_f Sin\frac{k\pi ct}{L} - \frac{k\pi c}{L} Sin\beta_f t}{\beta_f\left(\beta_f^2 - \left(\frac{k\pi c}{L}\right)^2\right)}\right) \tag{3.35}$$

Which on inversion yields

$$V_n(x,t) = \sum_{m=1}^{n} \frac{P}{2\mu_0 B_{11}}\left(\frac{\beta_f Sin\frac{k\pi ct}{L} - \frac{k\pi c}{L} Sin\beta_f t}{\beta_f\left(\beta_f^2 - \left(\frac{k\pi c}{L}\right)^2\right)}\right) Sin\frac{m\pi x}{L} \tag{3.36}$$

Equation (3.36) represents the response of a moving force for non-uniform simply supported Bernoulli-Euler beam resting on bi-parametric sub-grades.

CaseII: Simply supported non-uniform Bernoulli-Euler beam transverse by moving mass.

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When we set $\forall \neq 0$ in equation (3.15), we have the moving mass problem written as

$$\ddot{W}_m(t) + \beta_f^2 W_m(t) + \frac{\forall L}{B_{11}}[B_{13}\ddot{W}_m(t) + B_{14}\dot{W}_m(t) - B_{15}W_m(t)] = P\sin\frac{k\pi ct}{L} \tag{3.37}$$

Rearrangement of (3.37) to obtain

$$\ddot{W}_m(t) + \frac{2cm\pi L\forall R_{12}}{B_{11}\left(1+\frac{L\forall R_{11}}{B_{11}}\right)}\dot{W}_m(t) - \left(\frac{\frac{c^2m^2\pi^2\forall R_{11}}{B_{11}.L} - \beta_f^2}{1+\frac{\forall LR_{11}}{B_{11}}}\right)W_m(t) = \frac{\forall LgSin\frac{k\pi ct}{L}}{B_{11}\left(1+\frac{\forall LR_{11}}{B_{11}}\right)} \tag{3.38}$$

but

$$\frac{1}{1+\frac{\forall LR_{11}}{B_{11}}} = 1 - \frac{1}{1+\frac{\forall LR_{11}}{B_{11}}} + 0(\forall) \tag{3.39}$$

By using (3.39) into (3.30), we obtains

$$\begin{aligned}&\ddot{W}_m(t) + \frac{2cm\pi\forall L}{B_{11}}\dot{W}_m(t) - \frac{c^2m^2\pi^2\forall R_{11}}{B_{11}L}W_m(t)\\ &+\beta_f^2(1-\frac{\forall LR_{11}}{B_{11}})W_m(t) = \frac{\forall Lg}{B_{11}}Sin\frac{k\pi ct}{L}\end{aligned} \tag{3.40}$$

Where terms to $0(\forall^2)$ are neglected.

We now employed struble's method to get the modified frequency corresponding to the frequency of the free system due to the presence of the moving mass. An equivalent free system operator defined by the modified frequency then replaces equation (3.40). We set the right hand side of (3.40) to zero and a parameter $\forall_0 < 1$is considered for any arbitrary ratio $\forall$, defined as

$$\forall_0 = \frac{\forall}{1+\forall} \tag{3.41}$$

So that

$$\forall = \forall_0 + 0\left(\forall_0^2\right) \tag{3.42}$$

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Using (3.42) in (3.40), we have

$$\ddot{W}_m(t) + \frac{2cm\pi\forall_0 L}{B_{11}}\dot{W}_m(t) - \frac{c^2m^2\pi^2\forall_0 R_{11}}{B_{11}L}W_m(t) + \beta_f^2(1 - \frac{\forall_0 LR_{11}}{B_{11}})W_m(t) = \frac{\forall_0 Lg}{B_{11}} Sin\frac{k\pi ct}{L} \tag{3.43}$$

We assume a solution to the homogeneous part of (3.43) to be of the form

$$W_m(t) = \wedge(m,t)Cos\left(\beta_f t - \theta(m,t)\right) + \forall_0 W_1(m,t) + 0\left(\forall_0^2\right) \tag{3.44}$$

Using (3.44) with its derivatives in homogeneous part of (3.43) while neglecting terms of $0(\forall_0^2)$, we have

$$-2\dot{\Lambda}(m,t)\beta_f Sin[\beta_f t - \theta(m,t)] + \Lambda(m,t)\dot{\theta}(m,t)\beta_f Cos[\beta_f t - \theta(m,t)] - \frac{c^2m^2\pi^2\forall_0}{2B_{11}L}\Lambda(m,t)Cos[\beta_f t - \theta(m,t)] - \frac{\beta_f^2\forall_0 L}{2B_{11}}\Lambda(m,t)Cos[\beta_f t - \theta(m,t)] \tag{3.45}$$

By equating the coefficient of $Sin[\beta_f t - \theta(m,t)]$ *and* $Cos[\beta_f t - \theta(m,t)]$ to zero, we have the variational equations as

$$-2\dot{\Lambda}(m,t)\beta_f = 0 \tag{3.46}$$

$$2\dot{\Lambda}(m,t)\dot{\theta}(m,t) - \frac{C^2m^2\pi^2\forall_0\Lambda(m,t)}{2B_{11}L} - \frac{\beta_f^2\forall_0 L\Lambda(m,t)}{2B_{11}} = 0 \tag{3.47}$$

From (3.46), we have

$$\Lambda(m,t) = C^0 \tag{3.48}$$

where C^0 is a constant.

Also from (3.48), we get

$$\theta(m,t) = \left(\frac{c^2m^2\pi^2 + \beta_f^2L^2}{4\beta_{11}\beta_f L}\right)\forall_0 t + \theta_m \tag{3.49}$$

where θ_m is a constant.

Substituting (3.48) and (3.49) in (3.44), we have the first approximation to the homogeneous system as

$$W_m(t) = C^0 Cos(\alpha_f t - \theta_m) \tag{3.50}$$

Where

$$\alpha_f = \beta_f - \left(\frac{c^2 m^2 \pi^2 + \beta_f^2 L^2}{4\beta_{11}\beta_f L}\right)\forall_0 \tag{3.51}$$

Is the modified frequency of the free system due to the presence of the moving mass. The non-homogeneous equation (3.43) is solved by replacing the differential operator which acts on $W_m(t)$ with the equivalent free system operator defined by the modified frequency, α_f that is;

$$\frac{d^2W(t)}{dt^2} + \alpha_f^2 W_m(t) = \frac{\forall_0 Lg}{B11} Sin\frac{k\pi ct}{L} \tag{3.52}$$

Equation (3.52) is analogous to equation (3.33). We thus infer its solution as

$$W_m(t) = \frac{\forall_0 Lg}{2\alpha_f B_{11}}\left(\frac{\alpha_f Sin\frac{k\pi ct}{L} - \frac{k\pi c}{L} Sin\alpha_f t}{\alpha_f^2 - (\frac{k\pi c}{L})^2}\right) \tag{3.53}$$

with inverse;

$$V_n(x,t) = \sum_{m=1}^{n} \frac{\forall_0 Lg}{2\alpha_f B_{11}}\left[\frac{\alpha_f Sin\frac{k\pi c}{L}t - \frac{k\pi c}{L} Sin\alpha_f t}{\alpha_f^2 - (\frac{k\pi c}{L})^2}\right] Sin\frac{m\pi x}{L} \tag{3.54}$$

5. ANALYSIS OF RESULTS

The response amplitude of a dynamical system such as this may grow without bound. Conditions under which this happens are termed resonance conditions. Evidently, from equation (3.36), the non-uniform Bernoulli-Euler beam response under a moving force will grow witnout bound whenever.

$$\beta_f = \frac{k\pi c}{L} \tag{5.1}$$

while from equation (3.54), the same Bernoulli-Euler beam traversed by a moving mass encounter a resonance effect at

$$\alpha_f = \frac{k\pi c}{L} \tag{5.2}$$

From equation (3.51) we have

$$\alpha_f = \beta_f - \left(\frac{c^2 m^2 \pi^2 + \beta_f^2 L^2}{4 B_{11} \beta_f L} \right) \forall_0 \tag{5.3}$$

It can be deduced from equation (5.3) that, for the same natural frequency, the critical speed for the system of Bernoulli-Euler beam traversed by a moving mass is smaller than that of the same system traversed by a moving force. Thus, for the same natural frequency of the Bernoulli-Euler beam, the resonance is reach earlier by considering the moving mass system than by moving force system.

6. NUMERICAL CALCULATIONS AND DISCUSSIONS OF RESULTS

In this section, numerical results for the non-uniform simply supported Bernoulli-Euler beam are presented in plotted curves. An elastic beam of length 12.192m is considered. Other values used are modulus of elasticity E = 2.10924 x 10^{10}N/m^2, the moment of inertia I = 2.87698 x 10^{-3}m and mass per unit length of the beam μ = 3401.563Kg/m. The value of the foundation constant *(k)* is varied between *ON/m^3 and 400000N/m^3*, the value of axial force *N* is varied between *ON* and 2.0 x 10^8N, the values of the shear modulus (G) varied between *ON/m$^{3\ and}$ 900 000N/m^3*. The results are as shown in the various graphs below.

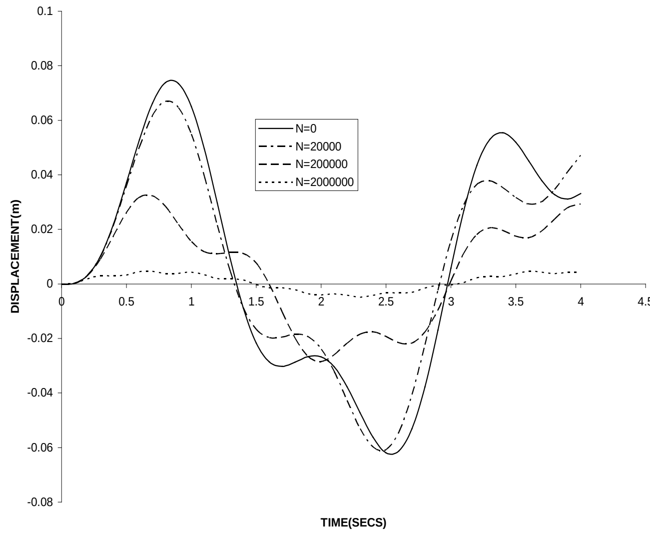

Fig 6.1: Deflection profile of a Simply Supported Non-Uniform Bernoulli-Euler Beam under moving force for fixed values of Shear modulus (G=90000), Foundation Modulus (k=40000) and various values of Axial Force (N)

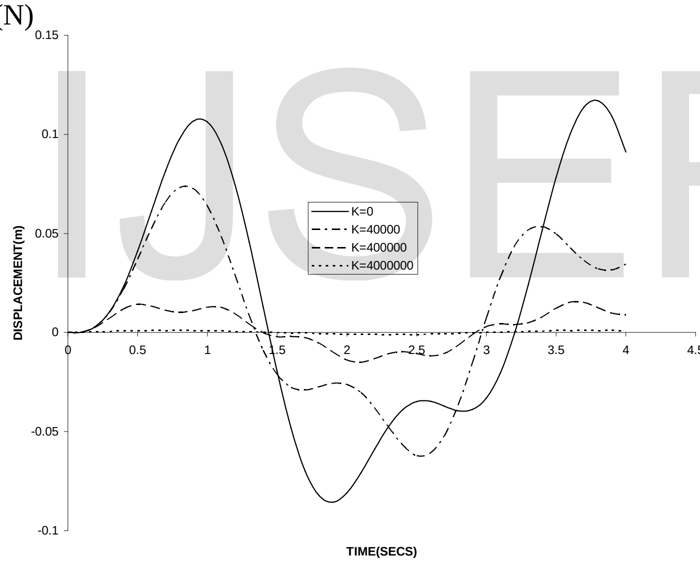

Fig 6.2: Deflection profile of a Simply Supported Non-Uniform Bernoulli-Euler Beam under moving force for fixed values of Shear modulus (G=90000), Axial force (N=20000) and various values of Foundation modulus (K)

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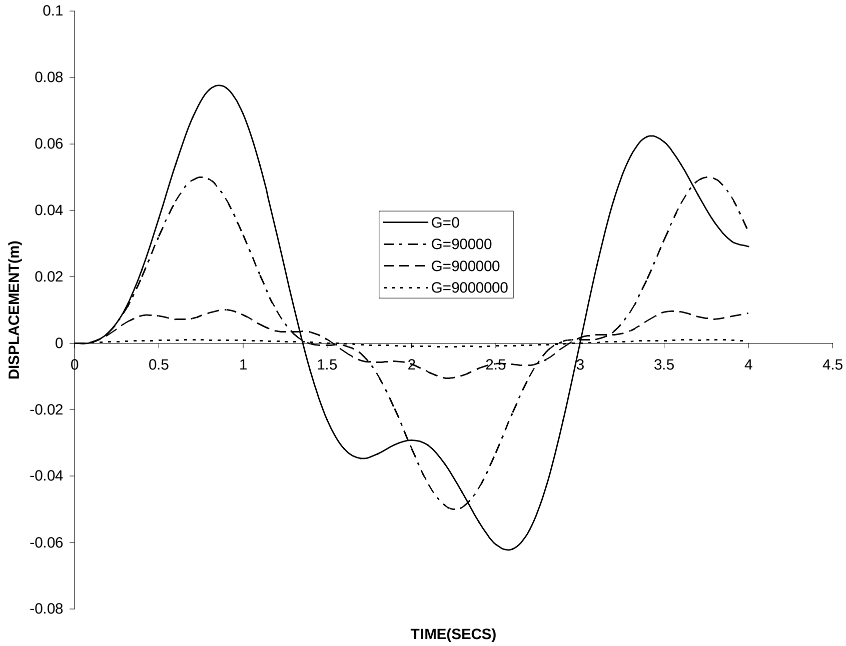

Fig 6.3: Deflection profile of Simply Supported Non-Uniform Bernoulli Euler Beam Transverse by Moving force for fixed value of Axial Force (N=20000), Foundation Modulus (G=40000) and various values of Shear Modulus (G)

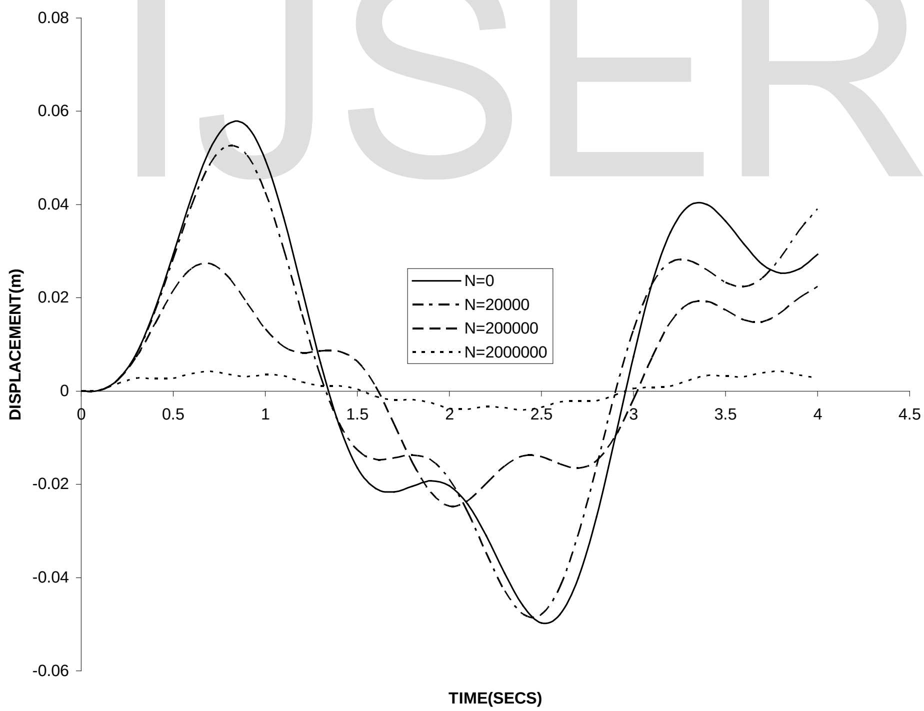

Fig 6.4: Deflection profile of a Simply Supported Non- Uniform Bernoulli-Euler Beam under moving mass for fixed values of Shear modulus (G=90000), Foundation Modulus (k=40000) and various values of Axial Force (N).

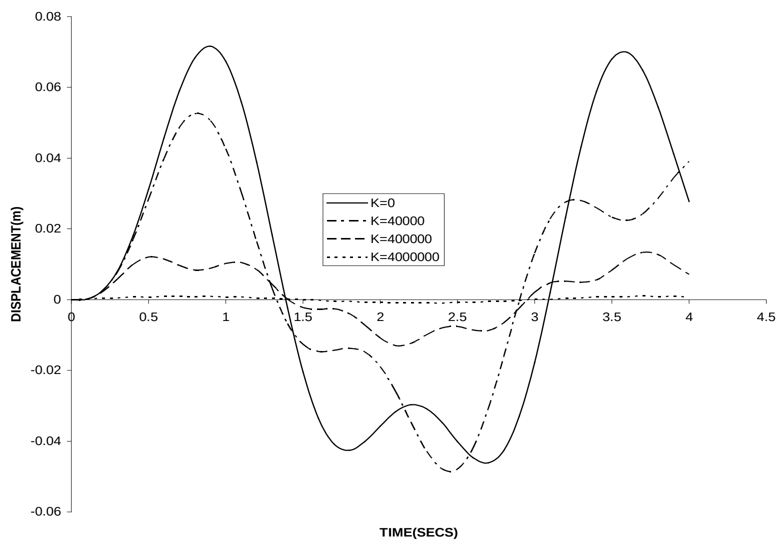

Fig 6.5: Deflection profile of a Simply Supported Non-Uniform Bernoulli-Euler Beam under moving mass for fixed values of Shear modulus (G=90000), Axial force (N=20000) and various values of Foundation modulus (K)

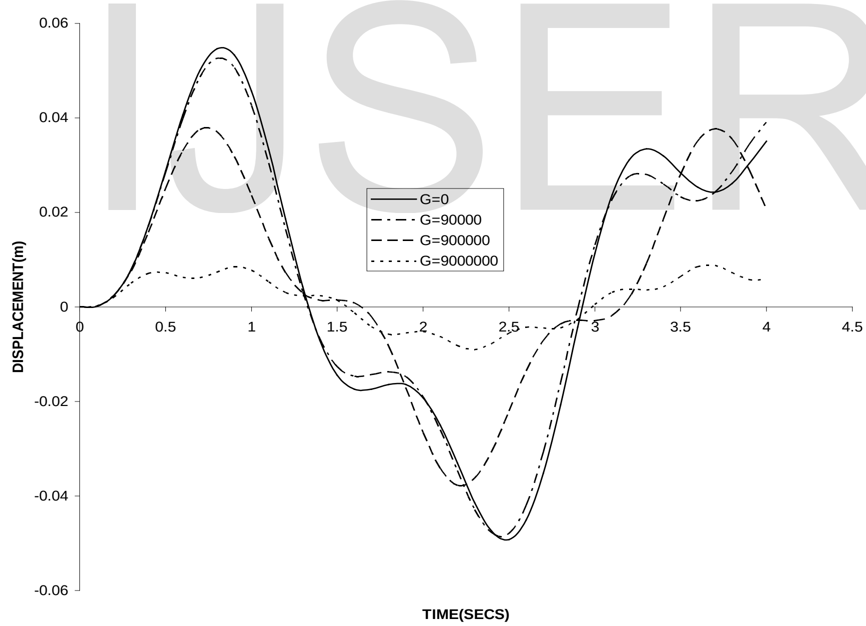

Fig 6.6: Deflection profile of Simply Supported Non-Uniform Bernoulli Euler Beam Transverse by Moving mass for fixed value of Axial Force (N=20000), Foundation Modulus (G=40000) and various values of Shear Modulus (G)

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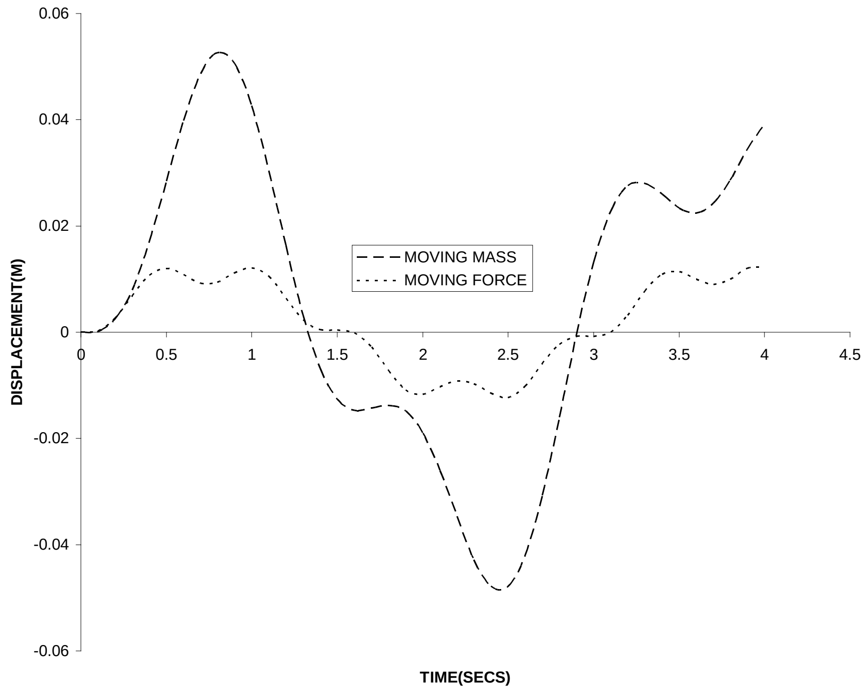

Fig (6.7): Comparison of the Deflection profile of moving force and moving mass cases of Simply Supported Non-Uniform Bernoulli-Euler Beam with foundation modulus (K=400000), Shear Modulus (G=90000) and Axial Force (N=20000).

7. Conclusion

In this paper, the problem of the dynamic response to moving concentrated load of a prestressed non-Uniform Simply Supported Bernoulli-Euler beam resting on bi-parametric subgrades, in particular, Pasternak subgrades has been solved. The approximate analytical solution technique is based on the Galerkin's method, Laplace transformation and convolution theory and finally modification of the Struble's asymptotic method. Analytical solutions and Numerical analysis show that, the critical speed for the same system consisting of a pre-stressed non-uniform simply supported Bernoulli-Euler beam resting on bi-parametric subgrades, in particular, Pasternak subgrades and traversed by a moving mass is smaller than that traversed by a moving force and this shows that, moving force solution is not an upper bound for the accurate solution of the moving mass problem. Furthermore, an increase in the foundation modulus K with fixed values of shear modulus G and axial force N reduces the amplitudes of vibration of the beam. Also, the amplitudes of vibration decreases with an increases in the values of the shear modulus with fixed values of foundation modulus and axial force. Also, increase in the values of the axial force with fixed values of shear modulus and foundation modulus. Finally, it was observed that, higher values of shear modulus are required for a more noticeable effect than that of the foundation modulus.

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